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On the non-identification of counterfactuals in dynamic discrete games

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In single-agent dynamic discrete choice models, counterfactual behavior is identified for some (but not all) counterfactuals despite the fact that the models themselves are under-identified. We review recent results on the identification of counterfactuals in dynamic discrete choice settings. When it comes to dynamic discrete games, we argue that counterfactuals are not identified, even when analogous counterfactuals of single-agent models are identified. Using the example of a duopoly entry game, we explain why strategic considerations undermine the identification of counterfactual equilibria in dynamic games.

1. Introduction

Although the main reason researchers estimate structural econometric models is to perform counterfactual simulations, we understand much less about the identification of counterfactuals than we do about the identification of econometric models themselves.

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Sometimes, counterfactuals of interest may be fully identified even when the associated model is not. This is the case for dynamic discrete choice (DDC) models, which have been applied in a variety of contexts (e.g. labor markets, firm dynamics, health choices). DDC models are typically underidentified, meaning that researchers must make restrictions on the model in order to estimate it. However, the non-identification of DDC models does not necessarily imply the non-identification of counterfactual behavior. In this paper, we review recent results exploring when counterfactual behaviour of dynamic discrete choice models are identified. While such results have been developed for the case of single-agent models, they have strong implications for the identification of counterfactuals of dynamic games. We present an important implication in the context of a dynamic duopoly entry game and explain why strategic considerations can undermine the identification of counterfactual equilibria in games.

Rust (1994), Magnac and Thesmar (2002), and Pesendorfer and Schmidt-Dengler (2008) showed that DDC models, under very general assumptions, are nonparametrically not identified: there are always many different utility functions that can rationalize observed choice behavior. Given this fact, Heckman and Navarro (2007) claim that "the entire dynamic discrete choice project thus appears to be without empirical content, and the evidence from it at the whim of investigator choices about functional forms of estimating equations and application of ad hoc exclusion restrictions."

Whimsically imposed or not, the restrictions that allow researchers to estimate a DDC model might not matter for the counterfactuals the researchers are ultimately interested in. Counterfactuals typically involve a change in utility functions, in the process governing state transitions, and/or in the set of actions and states available to agents. While there are always many utility functions consistent with the observed data, if each of those utility functions generates the same behavioral response to a given counterfactual, then that counterfactual can be said to be identified. In other words, identified counterfactuals are not sensitive to restrictions that are necessary to identify the model. Recent work has shown that some (but not all) counterfactuals of single-agent dynamic models are indeed identified. Aguirregabiria (2010), Norets and Tang (2014), Aguirregabiria and Suzuki (2014), and Arcidiacono and Miller (2015) consider some examples of counterfactuals. Kalouptsidi et al. (2016)) (henceforth KSS) offer instead a full characterization (necessary and sufficient conditions) of a very broad class of counterfactuals encompassing the previous examples. In particular, our results apply to counterfactuals which may simultaneously alter payoffs and transitions, as well as the choice set and state space. We consider models with nonparametric payoffs as well as commonly used parametric models, and we consider the identification of welfare changes.

After reviewing some of these recent results, we investigate how they extend to dynamic games. We consider a duopoly entry game and the firms' counterfactual response to a change in entry costs. While the change in question produces an identified response in a single-agent context (or if the behavior of one of the firms was held fixed), strategic considerations prevent the identification of the firms' responses. In other words, the counterfactual equilibrium is sensitive to necessary identifying restrictions imposed on the game's payoff function. Lack of identification is not the result of multiplicity of equilibria; a counterfactual equilibrium for one payoff function consistent with observed data will not be an equilibrium for another payoff function also consistent with the observed data. Given this result, we conjecture that strategic considerations imply that all practically relevant counterfactuals of dynamic discrete games will fall into a class of counterfactuals that are not identified.

The rest of the paper is organized as follows. Section 2 provides an overview of the literature on the identification of payoffs and counterfactuals in dynamic discrete choice models. Section 3 considers the example of a duopoly entry game.

2. Identification in dynamic single agent models

In this section, we review the standard dynamic discrete choice modeling framework and summarize some recent results on the identification of counterfactuals in single-agent models.

In the standard dynamic discrete choice framework, an agent i chooses one action a_{it} from the finite set $\mathcal{A} = \{1, \ldots, A\}$ in each period $t \in \{1, 2, \ldots\}$. The current payoff depends on the state variables $(x_{it}, \varepsilon_{it})$, where x_{it} is observed by the econometrician and ε_{it} is not. We assume $x_{it} \in \mathcal{X} = \{x_1, \ldots, X\}$, $X < \infty$; while $\varepsilon_{it} = (\varepsilon_{it}(1), \ldots, \varepsilon_{it}(A))$ is i.i.d. across agents and time and has joint distribution G. The transition distribution function for $(x_{it}, \varepsilon_{it})$ factors as

$$F(x_{it+1}, \varepsilon_{it+1} | a_{it}, x_{it}, \varepsilon_{it}) = F(x_{it+1} | a_{it}, x_{it}) G(\varepsilon_{it+1}),$$

and the current utility function is given by

$$u(a, x_{it}, \varepsilon_{it}) = u(a, x_{it}) + \varepsilon_{it}(a).$$

Agent i chooses a sequence of actions to maximize the expected discounted payoff. Given these assumptions, agent i's Bellman equation is

$$V(x_{it}, \varepsilon_{it}) = \max_{a \in \mathcal{A}} \{u(a, x_{it}) + \varepsilon_{it}(a) + \beta E[V(x_{it+1}, \varepsilon_{it+1}) | a, x_{it}]\},$$

where $\beta \in (0,\ 1)$ is the discount factor. The conditional choice probabilities (CCPs), $p_a(x_{it})$, are given by the probability of choosing action $a \in \mathcal{A}$ at time t conditional on $x_{it} \in \mathcal{X}$.

Researchers typically have access to panel data on actions and states. From such data, it is straightforward to construct estimates of CCPs and transition probabilities; e.g., with sufficiently rich data, simple frequency estimates suffice. Studies on identification typically take for granted that CCPs and transition probabilities are known and then consider what can be inferred about the model primitives $(u,\ \beta,\ G)$ or endogenous functions of the model primitives, such as counterfactual CCPs and welfare.

As previously noted, Rust (1994), Magnac and Thesmar (2002), and Pesendorfer and Schmidt-Dengler (2008) showed that payoffs in single-agent dynamic discrete choice models are nonparametrically not identified, even when (β, G) is known. In particular, Magnac and Thesmar (2002) characterized the degree of underidentification: the set of restrictions a DDC model imposes on data results in a system of equations that has infinitely many solutions. Indeed, since the researcher only has $(A-1)\times X$ linearly independent CCP estimates to identify the $A \times X$ elements in the utility function, the dimension of the set of solutions of the system is given by the cardinality of the state space, X.

In order to obtain point identification of u, one needs to add extra restrictions. Common extra assumptions include (combinations of) parametric functional forms, exclusion restrictions, and "normalizations." Parametric assumptions reduce the number of parameters to be identified and impose some shape restrictions; e.g., payoffs that are linear in observable states. Exclusion restrictions assume that some payoffs do not depend on all state variables; e.g., firm entry and exit costs are often assumed state invariant. "Normalizations" fix the payoff of some action in some states at some known value. Typically, the payoff of the action we have least information about is set to zero; usually referred to as the "outside option." Note that "normalizing" the utility of one action to some pre-specified level is different from imposing a normalization in the traditional sense. For instance, if we take action J and set u_J equal to a zero vector, we implicitly assume that the payoffs to action J do not depend on the state − that is a substantive assumption, and not something that can be arrived at through a positive transformation of the utility function.

Nonidentification of flow utilities seemingly poses serious challenges for counterfactual analysis. If different restrictions imposed on the model can lead to different utility functions which are both equally consistent with the observed data, then it might seem that the output of DDC models is necessarily sensitive to restrictions imposed by researchers. However, both of these models might agree in how the agents respond to a given policy intervention. If all models consistent with the observed data agree on the response to a given counterfactual change, then we can say that the counterfactual in question is identified.

In KSS, we offer a full characterization (necessary and sufficient conditions) of a broad class of counterfactuals, laying out conditions that can be checked in practice to verify whether an arbitrary counterfactual is identified. Our results apply to counterfactuals involving *both* changes in utility functions and transition matrices, nonlinear transformations of the payoff function, changes in the set of actions from the choice set as well as in the set of state variables. We also consider the identification of welfare changes and how the class of identified counterfactuals expands when the utility function has popular parametric restrictions.

To be specific, counterfactuals consist of transformations of model primitives, notably the set of actions $(\mathcal{A})$ and states $(\mathcal{X})$, the utility functions (u), and the transition probabilities (F). A counterfactual that changes payoffs u to $\widetilde{u}$, is described by a known function, h: $R^{AX} \rightarrow R^{AX}$ (so that $\widetilde{u} = h(u)$). A counterfactual can also change transitions F to $\widetilde{F}$. For the purposes of this paper, we consider only the following two results from the literature (Aguirregabiria, 2010; Aguirregabiria and Suzuki, 2014; Kalouptsidi et al., 2016; Norets and Tang, 2014), which are useful to understand identification of counterfactuals in the context of dynamic games.

Result 1. In a single-agent setting, if $\widetilde{F} = F$ and $\widetilde{u} = u + g$, where g is a known vector, then counterfactual choice probabilities $\widetilde{p}$ are identified.

In interpreting Result 1, it is important to note that the change in utility g is flexible in one way, but quite limited in another. It is flexible in the sense that it allows for changes in utility to be of any finite size and conditioned arbitrarily on actions and states. This seemingly allows for arbitrary changes in the payoff function, but g is not allowed to depend on u; thus, the researcher must be able to specify g before estimating the model. KSS denote this type of hypothetical change an “additive transfers” counterfactual.

Result 2. In a single-agent setting, if a counterfactual changes the transition process from F to $\widetilde{F}$, but the utility function is unchanged, then counterfactual choice probabilities $\widetilde{p}$ are not identified unless

$$(I - \beta F_a)(I - \beta F_J)^{-1} - \left(I - \beta \widetilde{F}_a\right)\left(I - \beta \widetilde{F}_J\right)^{-1} = 0 \tag{1}$$

for all $a \neq J$, where $F_a \in \mathbb{R}^{X \times X}$ is the transition matrix conditional on action a, and I is an identity matrix of size X.

Result 2 says that counterfactuals in which the transition matrix changes are typically not identified except in knife-edge cases. Together, Results 1 and 2 say that some counterfactuals are identified and some are not, but there exists many more possible counterfactuals that these results do not cover. The reader is referred to KSS for other types of counterfactuals.

Result 2 is even more pessimistic when extended to dynamic games. In a dynamic game, the transition process faced by an individual agent typically depends on the behavior of other agents – i.e., the transition matrices of dynamic games are not merely exogenous objects. Thus, even if a counterfactual of a dynamic game does not involve a change in the primitives of the transition process, it will typically involve changes in the transition process faced by individual agents. Section 3 expands on this idea in greater detail.

3. A dynamic entry game

In this section, we consider a simple example to help explain why counterfactual equilibria of dynamic games are likely non-identified even when analogous counterfactuals of single-agent models are identified.

Consider a duopoly entry game with two players indexed by $i = A, B$. In each period t, the players simultaneously choose whether to be active in the market ($a_{it} = 1$) or not ($a_{it} = 0$). Associated with each player is a state variable which equals the player's action in the previous period: $s_{it} = a_{i,t-1}$. The state of the game is simply the pair of states, $x_t = (s_{At}, s_{Bt})$, which is common knowledge. The game is symmetric, and player i receives payoffs which may depend on her own action, on her opponent's action and on the state variables. The current payoff function is given by

$$u(a_{it}, a_{-it}, x_{it}, \varepsilon_{it}) = u(a_{it}, a_{-it}, x_{it}) + \varepsilon_{it}(a_{it})$$

where $\varepsilon_{it} = (\varepsilon_{it}(0), \varepsilon_{it}(1))$ is private information of agent i, i.i.d. across agents and time, and has a joint normal distribution with variance $1/2$.[1] We assume ε_{it} is not observed by the econometrician and its joint distribution is common knowledge among players. To abstract away from problems related to multiplicity of equilibria, we impose a selection rule that always picks symmetric equilibria. Pesendorfer and Schmidt-Dengler (2008) show that a symmetric Markov perfect equilibrium exists in this context (see their Corollary 1).

Absent extra restrictions, there are sixteen different combinations of $(a_i, a_{-i}, s_i, s_{-i})$ and therefore the payoff function's parameter space is potentially sixteen-dimensional. However, an equilibrium of the game will only involve up to eight linearly independent choice probabilities (one for each player and state). As previously mentioned, we consider only symmetric equilibria for simplicity, and symmetric equilibria only involve four linearly independent choice probabilities. Thus, in a symmetric equilibrium we are only able to identify four parameters. Whether we focus on symmetric equilibria or not, we need extra restrictions on the payoff function to identify the *model*.

The question of interest in this note however focuses on whether we need restrictions in order to identify *counterfactual behavior*. Perhaps any restrictions we might make in order to identify the model will lead to the same results when we simulate counterfactual behavior; i.e., perhaps some counterfactuals are identified even though the model is not.

As a baseline case, we consider the following payoff function:

$$u_i(a_i, a_{-i}, s_i, s_{-i}) = \begin{cases} 0 & \text{if } a_i = 0, s_i = 0 \\ \phi & \text{if } a_i = 0, s_i = 1 \\ \pi_1 - c & \text{if } a_i = 1, s_i = 0, a_{-i} = 0 \\ \pi_1 & \text{if } a_i = 1, s_i = 1, a_{-i} = 0 \\ \pi_2 - c & \text{if } a_i = 1, s_i = 0, a_{-i} = 1 \\ \pi_2 & \text{if } a_i = 1, s_i = 1, a_{-i} = 1 \end{cases} \tag{2}$$

If the firm stays out of the market, it receives zero. When it enters, it pays an entry cost c and receives either the monopolist profit π_1 or the duopoly profit π_2. If the firm was

[1] This game was introduced by Pesendorfer and Schmidt-Dengler (2008). Equivalently, we can ignore the $\varepsilon_{it}(1)$ shock and assume $\varepsilon_{it}(0)$ is standard normal as they do.

Table 1
Entry game: payoff functions.

$\pi(a_i, a_i, s_i, s_{-i})$		Model 1				Model 2			
		$(a_i, a_{-i}) =$				$(a_i, a_{-i}) =$			
s_i	s_{-i}	(0, 0)	(0, 1)	(1, 0)	(1, 1)	(0, 0)	(0, 1)	(1, 0)	(1, 1)
0	0	0	0	1	−1.4	0	0	0	−0.506
0	1	0	0	1	−1.4	0	0	0	−1.11
1	0	0.1	0.1	1.2	−1.2	0	0	0	1.51
1	1	0.1	0.1	1.2	−1.2	0	0	0	−0.399

Table 2
Entry game: choice probabilities.

$P(\text{active}\|s_i,\ s_i)$		Baseline	CF – fixed opponent		CF – equilibrium	
			Model 1	Model 2	Model 1	Model 2
s_i	s_{-i}					
0	0	0.576	0.527	0.527	0.634	0.516
0	1	0.305	0.257	0.257	0.207	0.243
1	0	0.842	0.875	0.875	0.983	0.836
1	1	0.595	0.643	0.643	0.692	0.612

already active, it receives either π_1 or π_2. Finally, if the firm decides to exit, it receives the scrap value ϕ. Note that the above payoff function represents a restrictive parameterization. In principle, payoffs when a player exits ($a_{it} = 0$ and $s_{it} = 1$) might depend on the other player's behavior and/or state variable. The baseline parameterization is $\phi = .1$, $c = .2$, $\pi_1 = 1.2$, and $\pi_2 = -1.2$.

We refer to the model with the payoff function described above as Model 1 (the true model); Table 1 specifies the payoffs for each combination of actions and states. Table 2 describes a symmetric equilibrium for this model. In this equilibrium, each player enters with probability .576 when both players were not active in the previous period. Each player remains active with probability .595 when both players competed in the previous period. When only one player was active in the previous period, the incumbent remains active with probability .842 and the other firm enters with probability .305.

Given the under-identification of the model, there are many other payoff functions that could rationalize this baseline equilibrium. One such alternative is Model 2, where we restrict all payoffs to be zero except when $(a_i, a_{-i}) = (1, 1)$:

$$u_i(a_i, a_{-i}, s_i, s_{-i}) = \begin{cases} 0 & \text{if } a_i = 0 \text{ or } a_{-i} = 0 \\ \pi_{00} & \text{if } a_i = 1, a_{-i} = 1, s_i = 0, s_{-i} = 0 \\ \pi_{01} & \text{if } a_i = 1, a_{-i} = 1, s_i = 0, s_{-i} = 1 \\ \pi_{10} & \text{if } a_i = 1, a_{-i} = 1, s_i = 1, s_{-i} = 0 \\ \pi_{11} & \text{if } a_i = 1, a_{-i} = 1, s_i = 1, s_{-i} = 1 \end{cases} \quad (3)$$

Table 1 also describes parameter values for (3) which rationalize the baseline equilibrium. One may view these values as the parameter estimates that an econometrician would obtain by imposing specification (3) when using a data set generated by Model 1.

While the baseline equilibrium is an equilibrium of both Models 1 and 2, it is not obvious a priori whether the equilibria of the two models will coincide when the payoff functions are changed. We consider a counterfactual in which entry costs are increased by .25 (in levels, not in proportional terms). Formally, we consider the following transformation of the payoff function for each of the two models:

$$\widetilde{u}(a_i, a_{-i}, s_i, s_{-i}) = \begin{cases} u(a_i, a_{-i}, s_i, s_{-i}) - .25 & \text{if } a_i = 1, s_i = 0 \\ u(a_i, a_{-i}, s_i, s_{-i}) & \text{otherwise} \end{cases}$$

The counterfactual payoffs $\widetilde{u}$ increase the costs of entry relative to the original payoffs u. In the notation of the previous section: $\widetilde{u} = u + g$, where $g(a_i, a_{-i}, s_i, s_{-i}) = -.25$ for $a_i = 1, s_i = 0$, and $g(a_i, a_{-i}, s_i, s_{-i}) = 0$ otherwise. This is an "additional transfers" counterfactual.

Before considering equilibria of the counterfactual games, it is helpful to first consider how an individual player's best response would change if her payoff function changed from u to $\widetilde{u}$ and her opponent's behavior remained fixed in the baseline equilibrium strategy. These best responses are described by the "CF – fixed opponent" columns of Table 2. They are identical for the two models – i.e., the different ways of rationalizing the baseline equilibrium are equivalent when we consider these interim best responses. This should not be surprising. As Result 1 tells us, additive transfers counterfactuals of single-agent models are identified, and this exercise could be described as an additive transfer of a single agent model. We can always look at the problem of solving for an individual player's best response as a single-agent problem, and if we hold the opponent's strategy fixed, then the only modification is the change in the payoff function from u to $\widetilde{u}$. The counterfactual $\widetilde{u}$ here involves only an additive transfer change, so the results from single-agent models apply, and we have identified interim best responses.

However, as the final two columns of Table 2 describe, the identification of interim best responses does not lead to identification of the counterfactual equilibrium. In the new equilibrium, the change from u to $\widetilde{u}$ is no longer the only change in player i's problem; player i must also consider the change in her opponent's strategy, and this amounts to a change in the transition process for the dynamic problem the player is solving. As Result 2 tells us, counterfactuals involving changed transition functions are not identified unless the very stringent condition (1) holds. Satisfying (1) in practice, however, seems unlikely.

For the particular parameterizations (2) and (3), Table 2 shows that, before strategic considerations are taken into account, Models 1 and 2 agree that increasing entry costs decreases the rate of entry regardless of whether the opponent is active or not. Reduced entry makes incumbent monopolists even more likely to remain active than in the baseline, for the risk of ending up in a duopoly is reduced.

On the other hand, the counterfactual predictions of the two models differ in the symmetric equilibrium. Under the true model (Model 1), it is not clear ex-antewhether increased entry costs will increase or decrease the probability of entry when both firms are inactive in the previous period. In the simulation, the appeal of becoming a monopolist increases the rate of entry when $(s_i, s_{-i}) = 0$. For Model 2, however, it decreases the rate of entry.

This is not the only difference observed in the counterfactual equilibrium. The identifying restrictions imposed by Model 2 result in a payoff function in which the firm earns the highest profits when both firms are active today but the opponent was inactive before. In this case, being a monopolist is not so appealing. The smaller appeal of being a monopolist lowers the rate at which incumbents stay active and also lowers further the rate at which firms enter when no firms are active. In summary, the counterfactual equilibrium under Model 2 is *qualitatively* very different from the counterfactual equilibrium under the (true) Model 1.

A formal investigation of the conditions under which counterfactuals are identified in dynamic games can be based on extensions of KSS. However, the set of conditions in KSS depend on properties of the transition process in both the baseline and counterfactual settings. For a single-agent model, checking the conditions amounts to checking primitives of the model because the transition process can be taken as a primitive. But for a dynamic game, the transition process is typically an equilibrium object, and therefore checking whether the conditions are satisfied is not simply a property of the primitives of the model. KSS's conditions cannot be checked for a dynamic game without first solving for counterfactual equilibria.

Yet, absent a formal investigation, the dynamic entry model suggests a reasonable conjecture. Counterfactuals of dynamic games typically involve strategic considerations, meaning that firms face changes in opponents' expected behavior. Changes in opponents' expected behavior imply that the transition process faced by a given agent changes. This interpretation of Result 2 suggests that counterfactuals of dynamic games are generically not identified.

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